Geometry Definitions, Postulates, and Theorems

Chapter 1: Essentials of Geometry

Section 1.1: Identify Points, Lines, and Planes

Standards: 1.0 Students demonstrate understanding by identifying and giving examples of undefined terms, axioms, theorems, and inductive and deductive reasoning.

**Three Undefined Terms in Geometry:** *Point, Line, & Plane.* Descriptions are below.

**Point** – Is represented by a small dot and symbolized with an upper case printed letter. Has no dimension. Has no weight, height, depth or length. Just has a location.

**Line** – Is represented by a straight line with two arrowheads to indicate that it extends without ending in two directions. Extends in one dimension. A line can be symbolized by two points, \( \overline{AB}, \overline{BA} \), or by a lower case script letter, \( l \). It is a series of points that go on forever.

**Plane** – Is represented by a four-sided shape that looks like a wall. The plane extends without ending. Extends in two dimensions. A plane can be symbolized by three points, \( \overline{ABC} \), not in a straight line, or by an upper case script letter, \( M \).

**Collinear** – Points that lie on the same line.

Ex. Points Q, R, and T are collinear.

Points Q, R, and S are noncollinear.

**Coplanar** – Points that lie on the same plane.

Ex. Name 3 points that are coplanar \( \Rightarrow \overline{LMN} \)

Name 4 points that are noncoplanar \( \Rightarrow \overline{LMNP} \)

Name 3 collinear points \( \Rightarrow \overline{JNO} \)

**Segment** – Symbolized \( \overline{AB} \).

The endpoints A and B and all points on the line that are between A and B. (part of a line).

Segment \( \overline{AB} \) or \( \overline{BA} \)
Ray – Symbolized $AB$.
The initial (starting) point, the first letter, $A$, and all points in the direction of the second letter, $B$.

Opposite Rays – Two rays with the same endpoint that make a straight line.

Intersect – The point(s) that two or more figures have in common.

Ex. Draw a line and a plane that do not intersect.

Ex. Draw 2 lines that intersect and one that does not.

Intersection – The set of points the figures have in common.

Ex. Draw a line and a plane that intersect. Their intersection forms a **Point**.

Ex. Draw 2 planes that intersect. Their intersection forms a **Line**.

Ex. Draw 3 collinear points $A$, $B$, $C$.
Draw $D$ which is noncollinear with $A$, $B$ and $C$.
Draw $AB$ and $BD$. 
Postulates or Axioms – Rules that are accepted without proof.

Coordinate – A real number that corresponds to a point on a number line.

-1 is the coordinate of point A

Distance of a segment – The absolute value of the difference between the two coordinates of the endpoints of the segment. Distance is also called length and is symbolized as $AB$. *Length/Distance must always be positive.

Ruler Postulate – The points on a line can be matched, one-to-one, with the real numbers.

Between – Applies to collinear points, where one point is between two others.

Segment Addition Postulate – IF $B$ is between $A$ and $C$, THEN $AB + BC = AC$. IF $AB + BC = AC$, THEN $B$ is between $A$ and $C$.

Congruent Segments – Segments that have the same length.

Written $\cong$
Section 1.3: Use Midpoint and Distance Formulas

Standards: Prepare for 7.0 Students prove theorems by using coordinate geometry, including the midpoint of a line segment, the distance formula, and various forms of equations of lines and circles.

**Midpoint** – A point, on a segment, that divides the segment into two congruent segments. The halfway point on a segment.

**Bisects** – Cutting, or dividing, a segment into two congruent segments.

**Segment Bisector** – A segment, a ray, a line, or a plane, that intersects a segment at the midpoint.

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Ex. M is the midpoint of segment $\overline{GH}$. Find MH.

$$
\begin{align*}
5x - 8 &= 3x + 14 \\
2x - 8 &= 14 \\
2x &= 22 \\
x &= 11 \\
\end{align*}
$$

$$
MH = 3x + 14
$$

$$
MH = 3(11) + 14
$$

$$
MH = 47
$$

**Midpoint Formula** – Given two ordered pairs, $(x_1, y_1)$ & $(x_2, y_2)$, you can calculate the midpoint by using the following formula:

$$
M = \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right). 
$$

*Think of it as the average of the $x$'s and average of the $y$'s

Ex. Find the coordinates of the midpoint of $\overline{AB}$ if A(-2,-3) and B(5,-2).

$$
M = \left( \frac{-2 + 5}{2}, \frac{-3 + (-2)}{2} \right)
$$

$$
M = \left( \frac{3}{2}, -\frac{5}{2} \right)
$$

Ex. The midpoint of $\overline{JK}$ is M(0, $\frac{1}{2}$).

One endpoint is J(2,-2), find K(x,y).

(over)
Distance Formula – Given two ordered pairs, \((x_1, y_1)\) & \((x_2, y_2)\), the distance is:

\[ d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \]

Ex. Given A(3,2) and B(-1,1), find AB.

\[
AB = \sqrt{(3-(-1))^2 + (2-1)^2} = \sqrt{16 + 1} = \sqrt{17}
\]

Ex. Find the distance between B (-1, 1) and C(-2, -1).

\[
BC = \sqrt{(-1 -(-2))^2 + (1 -(-1))^2} = \sqrt{1 + 4} = \sqrt{5}
\]

Pythagorean Theorem – In a right triangle, \((\text{Leg}_1)^2 + (\text{Leg}_2)^2 = (\text{Hypotenuse})^2\)

\[
a^2 + b^2 = c^2
\]

Review: Simplifying Radicals. (Doubles get you out of jail)

Ex. a) \(\sqrt{20}\)  
\[
\sqrt{20} = \sqrt{4 \cdot 5} = 2\sqrt{5}
\]

Ex. b) \(\sqrt{60}\)  
\[
\sqrt{60} = \sqrt{36 \cdot 1.5} = 6\sqrt{1.5}
\]

Ex. c) \(\sqrt{72}\)  
\[
\sqrt{72} = \sqrt{36 \cdot 2} = 6\sqrt{2}
\]

Ex. d) \(\sqrt{80}\)  
\[
\sqrt{80} = \sqrt{40 \cdot 2} = 4\sqrt{5}
\]

Ex. e) \(\sqrt{1050}\)  
\[
\sqrt{1050} = \sqrt{105 \cdot 10} = 5\sqrt{105}
\]
Section 1.4: Measure and Classify Angles

Standards: 1.0 Students demonstrate understanding by identifying and giving examples of undefined terms, axioms, theorems, and inductive and deductive reasoning.

**Angle** – Two rays that have the same initial point.

**Sides** – The rays of the angle. \( \overrightarrow{BA} \) & \( \overrightarrow{BC} \)

**Vertex** – The initial point \( B \)

An angle is symbolized by three letters, where the center letter is the vertex.

\( \angle ABC, \angle CBA, \angle B, \) or \( \angle 1 \)

**Measure of an angle** – Symbolized by \( m\angle ABC \).

Represents the number of degrees, \( \circ \), in an angle. A protractor is used to measure degrees.

*One protractor is 180 degrees
*two protractors together make a circle which is 360 degrees

**Protractor Postulate** - The rays of an angle can be matched one-to-one with the real numbers from 1 to 180, inclusive.

**Acute Angle** – An angle that measures *between* \( 0^\circ \) and \( 90^\circ \).

*****Right Angle** – An angle whose measure is *exactly* \( 90^\circ \).

**Obtuse Angle** – An angle that measures *between* \( 90^\circ \) and \( 180^\circ \).
**Straight Angle** – An angle whose measure is exactly 180°.

**Angle Addition Postulate** - IF $P$ is in the interior of $\angle RST$, THEN $m\angle RSP + m\angle PST = m\angle RST$.

**Congruent Angles** – Angles that have the same measure.

$\angle ABC = \angle CBD$ since $m\angle ABC = m\angle CBD$

Ex. Find $m\angle STV$.

Ex. Given $m\angle MNP = 70°$, find $m\angle MNO$.

Ex. Find the missing angle.

**Angle Bisector** – A ray that divides an angle into two congruent adjacent angles.

$\overline{HJ}$ bisects $\angle DHK$ if $\angle DHJ \cong \angle JHK$

or

$m\angle DHJ = m\angle JHK$

Ex. $\overline{CD}$ bisects $\angle ACB$. Given $m\angle ACD = (x+10)°$ and $m\angle BCD = (3x-20)°$, solve for $x$ and find $m\angle ACB$.

$m\angle ACB = x+10+3x-20$

$x+10 + 3x - 20 = 180$

$4x - 10 = 180$

$4x = 190$

$x = 47.5$

$m\angle ACB = 70°$
**Complementary Angles** – Two angles are complementary if the sum of their measures is $90^\circ$.

$\angle A$ and $\angle B$ are complementary. If $m\angle A = 20$, $m\angle B = 70$.

**Supplementary Angles** – Two angles are supplementary if the sum of their measures is $180^\circ$.

$\angle A$ and $\angle B$ are supplementary. If $m\angle A = 65$, $m\angle B = 115$.

**Linear Pair** – Two adjacent angles whose noncommon sides form opposite rays.

*The sum of the measures of a linear pair is $180^\circ$. 

\[
\begin{align*}
\angle 1 + \angle 2 &= 180^\circ \\
\angle 2 + \angle 3 &= 180^\circ \\
\angle 3 + \angle 4 &= 180^\circ \\
\angle 4 + \angle 1 &= 180^\circ
\end{align*}
\]
**Vertical Angles** – Opposite angles formed by two intersecting straight lines. *Vertical Angles are congruent.

Ex. True or False.

a) \( \angle 1 \) and \( \angle 2 \) are a linear pair. ________ True ________

b) \( \angle 4 \) and \( \angle 5 \) are a linear pair. ________ False ________

c) \( \angle 5 \) and \( \angle 3 \) are vertical angles. ________ False ________

d) \( \angle 1 \) and \( \angle 3 \) are vertical angles. ________ True ________

e) If \( m\angle 3 = 40^\circ \), find: \( m\angle 1 = 40 \) \( m\angle 2 = 140 \) \( m\angle 4 = 90 \) \( m\angle 5 = 50 \)

Ex. Given \( m\angle 1 = (4x + 15)^\circ \), \( m\angle 2 = (5x + 30)^\circ \), \( m\angle 3 = (3y - 15)^\circ \), and \( m\angle 4 = (3y+15)^\circ \), find \( x \) and \( y \) and the measure of each angle.

* Notice that the vertical angles are congruent.
Section 1.6: Classify Polygons

Standards: 12.0 Students find and use measures of sides and of interior and exterior angles of triangles and polygons to classify figures and solve problems.

**Polygon** – A plane figure that:
1. is formed by three or more segments called sides.
2. each side intersects exactly two other sides – one at each endpoint.

**Sides** – Segments connecting *consecutive* endpoints.

**Vertex** – Each endpoint of a side.

Ex. Is it a polygon? (write yes or no)

<table>
<thead>
<tr>
<th>a)</th>
<th>b)</th>
<th>c)</th>
<th>d)</th>
<th>e)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yes</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
</tr>
</tbody>
</table>

**Convex** – In a polygon, if the sides (segments) are extended in both directions (to make lines), then that line will not go through the interior of the polygon.

ENGLISH TRANSLATION – A polygon without any dents!

**Nonconvex (or concave)** – A polygon that is not convex.

ENGLISH TRANSLATION – A polygon with a dent, or two, or three, etc.!

**Polygons are named by the number of sides they have.**

<table>
<thead>
<tr>
<th>Number of sides</th>
<th>Type of polygons</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>triangle</td>
</tr>
<tr>
<td>4</td>
<td>quadrilateral</td>
</tr>
<tr>
<td>5</td>
<td>pentagon</td>
</tr>
<tr>
<td>6</td>
<td>hexagon</td>
</tr>
<tr>
<td>7</td>
<td>heptagon or septagon</td>
</tr>
<tr>
<td>8</td>
<td>octagon</td>
</tr>
<tr>
<td>9</td>
<td>nonagon</td>
</tr>
<tr>
<td>10</td>
<td>decagon</td>
</tr>
<tr>
<td>11</td>
<td>hendecagon</td>
</tr>
<tr>
<td>12</td>
<td>dodecagon</td>
</tr>
<tr>
<td>n</td>
<td>n-gon (“n” sides, where n is a variable)</td>
</tr>
</tbody>
</table>

(over)
**Regular** – A polygon that meets the following **TWO** conditions
1. The polygon is equilateral, all sides are congruent.
2. The polygon is equiangular, all angles are congruent.

**Diagonal** – A segment that joins two **nonconsecutive** vertices.

Ex. Draw a regular hexagon. Draw all the diagonals.

There are 6 sides and 9 diagonals.

* n = # of sides

Formula: \# of diagonals = \( \frac{n(n-3)}{2} = \frac{6(6-3)}{2} = \frac{18}{2} = 9 \)

Ex. Find the value of x in the regular polygon.
Section 1.7: Find Perimeter, Circumference, and Area

Standards: 8.0 Students know, derive, and solve problems involving the perimeter, circumference, area, volume, lateral area, and surface area of common geometric figures. 10.0 Students compute areas of polygons, including rectangles, scalene triangles, equilateral triangles, rhombi, parallelograms, and trapezoids.

**Perimeter** – Distance around the outside of a figure. Units are expressed in feet, centimeters, etc.

**Area** – How many squares fit inside a figure. Units are expressed in square units such as square feet, \( ft^2 \), square centimeters, \( cm^2 \), etc.

**Square** – 4 sided figure with 4 right angles and all sides equal.

\[
\text{Ex.} \quad \begin{array}{c}
3 \text{ cm} \\
\hline
3 \text{ cm}
\end{array}
\]

\[
\text{Perimeter} = 12 \text{ cm} \quad A = S^2
\]

**Rectangle** – 4 sided figure with 4 right angles and opposite sides equal.

\[
\text{Ex.} \quad \begin{array}{c}
8'' \\
5''
\end{array}
\]

\[
\text{Perimeter} = 26 \text{ in} \quad A = lw
\]

**Triangle** – 3 sided figure.

\[
\text{Ex.} \quad \begin{array}{c}
8 \text{ mm} \\
6 \text{ mm} \\
11 \text{ mm}
\end{array}
\]

\[
\text{Perimeter} = 28 \text{ mm} \quad A = \frac{1}{2}bh
\]

**Circle** – The set of all points at the same distance from a given point. \( A = \pi r^2 \)

**Radius** – A segment with one endpoint at the center and the other endpoint on the circle.

**Diameter** – A segment whose endpoints are on the circle and the segment goes through the center of the circle.

**Circumference** – The distance around the outside of a circle. \( C = 2\pi r \)

**Pi** – Symbolized \( \pi \). The ratio of a circle’s circumference to its diameter.
Ex. Find the area and perimeter of the triangle defined by $H(-2,2)$, $J(3,-1)$ and $K(-2,-4)$.

\[ HJ = \sqrt{(-2-3)^2 + (2+1)^2} = \sqrt{25 + 9} = \sqrt{34} \]

\[ JK = \sqrt{(3+2)^2 + (-1+4)^2} = \sqrt{25 + 9} = \sqrt{34} \]

$P = (6 + \sqrt{34} + \sqrt{34})$ units

$A = \frac{1}{2}(6)(5) = 15$ units$^2$

Ex. A rectangle has an area of $15.4 \text{ ft}^2$. It's length is $7 \text{ ft}$. What is the width?

\[ \frac{15.4}{7} = \frac{L \cdot W}{1} \]

\[ 2.2 \text{ ft} = W \]